

# Multi-phonon $\gamma$ -vibrational bands in odd-mass nuclei studied by triaxial projected shell model approach

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## Abstract

Inspired by the recent experimental data (Phys. Lett. B **675** (2009) 420), we extend the triaxial projected shell model approach to study the  $\gamma$ -band structure in odd-mass nuclei. As a first application of the new development, the  $\gamma$ -vibrational structure of  $^{103}\text{Nb}$  is investigated. It is demonstrated that the model describes the ground-state band and multi-phonon  $\gamma$ -vibrations quite satisfactorily, supporting the interpretation of the data as one of the few experimentally-known examples of simultaneous occurrence of one- and two- $\gamma$ -phonon vibrational bands. This generalizes the well-known concept of the surface  $\gamma$ -oscillation in deformed nuclei built on the ground-state in even-even systems to  $\gamma$ -bands based on quasiparticle configurations in odd-mass systems.

*Key words:* Multi-phonon  $\gamma$ -vibration, Odd-mass nuclei, Triaxial projected shell model approach

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Atomic nucleus is a many-body quantal system exhibiting pronounced shell effects, which give rise to intrinsic deformation. In addition, it can, according to the semiclassical collective model, undergo dynamical oscillations around the equilibrium shape, resulting in various low-lying collective excitations.

Ellipsoidal oscillation of the shape is commonly termed  $\gamma$ -vibration [1]. Rotational bands based on the  $\gamma$ -vibrational states are known as  $\gamma$ -bands. The interplay between rotational and vibrational degrees of freedom plays a central role in our understanding of structure of atomic nuclei. One-phonon  $\gamma$ -bands have been well known and observed in numerous deformed nuclei in most of the regions of the periodic table. However, observation of two or higher order phonon  $\gamma$ -bands is a rare event possibly because, due to excitation, these bands are embedded in the energy region with dense levels. Nevertheless, there have been reports on successful observation of two-phonon  $\gamma$ -band ( $\gamma\gamma$ -band) in well-deformed even-even nuclei [2,3]. Recently, fission experiment data have suggested simultaneous observation of one-phonon  $\gamma$ - and two-phonon  $\gamma\gamma$ -bands in odd-mass systems [4,5,6,7].

While the physics of one-phonon  $\gamma$ -bands seems to be well understood, there has been a considerable debate on the nature of  $\gamma\gamma$ -bands. The issue is whether these bands are really two-phonon excitations built on the rotational states or they are based on an ensemble of two-quasiparticle excitations. Theoretical investigations using quasiparticle-phonon nuclear model (QPNM) [8,9] predicted that  $\gamma\gamma$ -vibrational bands cannot exist in deformed nuclei due to Pauli blocking of quasiparticle components. On the other hand, the multi-phonon method (MPM) [10,11] suggested that a two-phonon  $K^\pi = 4^+$  state (in even-even nuclei) should appear at an excitation energy of about 2.6 times the energy of the one-phonon  $K^\pi = 2^+$  state, and the decay from the  $\gamma\gamma$ - to  $\gamma$ -band should be predominantly collective in character. Strictly speaking, due to the violation of rotational symmetry, these methods do not calculate the states of angular-momentum, but the  $K$  states (where  $K$  is the projection of angular-momentum on the intrinsic symmetry axis). Therefore, the QPNM and MPM models do not have their wave functions as eigenstates of angular-momentum, and consequently, the reliability of these predictions depends critically on the actual situation. As pointed out by Soloviev [9], it is quite desirable to recover good angular-momentum in the wave function.

Recently, the triaxial projected shell model (TPSM) approach has been developed and applied to investigate  $\gamma$ -bands in transitional even-even nuclei [12,13,14,15]. The TPSM approach [16], generalized form the one-dimensional projected shell model [17], performs explicit three-dimensional angular-momentum projection from triaxially deformed Nilsson states. Shell model diagonalization is carried out in the laboratory frame with good angular-momentum, thus removing uncertainties caused by angular-momentum non-conservation in the QPNM and MPM models.

In the TPSM, each triaxially-deformed configuration is an admixture of different  $K$  states. For example, the quasiparticle vacuum configuration is composed of  $K = 0, 2, 4, \dots$  states for an even-even system. It has been demonstrated in Ref. [12] that the angular-momentum-projection on these  $K = 0, 2$

and 4 vacuum states generates the low-spin parts of the ground,  $\gamma$ -, and  $\gamma\gamma$ -band, respectively. This is a pleasant feature of the TPSM that the quantum-mechanical treatment of angular-momentum-projection on the (triaxially deformed) quasiparticle vacuum alone gives rise to rotational ground band and multiphonon  $\gamma$ -vibrational bands. Of course, this simple configuration [12] is valid only for low-spin states because if the system goes to high spins, rotational alignment will bring various quasiparticle configurations down to the yrast region, and therefore, the quasiparticle excitations must be considered. In our recent work [14,15], the TPSM has been generalized to include two-neutron and two-proton quasiparticle states for even-even nuclei. It has been shown that indeed,  $\gamma$ -bands built on these quasiparticle states can become favoured at higher angular-momenta. Using this generalization, we have offered an explanation for the long-standing puzzle of the observation of two aligned  $I^\pi = 10^+$  states with same intrinsic neutron structure in the mass-130 region. It has been demonstrated [15] that  $\gamma$ -band built on the neutron aligned configuration is the first excited band above the parent aligned configuration, and this, together with the standard neutron aligned state itself, provides a natural explanation of the observation of two aligned structures with same intrinsic configuration.

In a very recent experimental paper [7],  $\gamma$ - and  $\gamma\gamma$ -bands have been reported in an odd-proton system,  $^{103}\text{Nb}$ , by measuring prompt  $\gamma$ -rays following the spontaneous fission of  $^{252}\text{Cf}$ . The ground state of this nucleus has the spin-parity  $K_g^\pi = 5/2^+$ . Based on this, the experimentally observed  $\gamma$ -band built on  $9/2^+$  and  $\gamma\gamma$ -band based on  $13/2^+$  have been proposed. Naively, the spin-2 phonon structure in an odd-mass nucleus is quite analogous to that in an even-even system based on the  $0^+$  ground state and one can have the  $\gamma$ -band built on  $K_\gamma = K_g + 2$  and  $\gamma\gamma$ -band based on  $K_{\gamma\gamma} = K_g + 4$ . However, there has been, so far, no theoretical work that describes this observation, and thus supporting the interpretation. The purpose of this Letter is to study these band structures using the newly developed TPSM approach.

The present study generalizes, for the first time, the TPSM to odd-mass nuclei with the inclusion of quasiparticle (qp) configurations in the model basis. For the study of odd-proton system, our model space is spanned by (angular-momentum-projected) one- and three-qp basis

$$\{\hat{P}_{MK}^I a_p^\dagger | \Phi \rangle, \hat{P}_{MK}^I a_p^\dagger a_{n1}^\dagger a_{n2}^\dagger | \Phi \rangle\}, \quad (1)$$

where the projector

$$\hat{P}_{MK}^I = \frac{2I+1}{8\pi^2} \int d\Omega D_{MK}^I(\Omega) \hat{R}(\Omega), \quad (2)$$

and  $| \Phi \rangle$  represents the triaxially-deformed qp vacuum state. The qp basis

chosen in (1) includes the configurations of two-neutron aligned states built on the one-quasiproton states. The basis, with one- and three-qp configurations included, has proven adequate to describe the high-spin states in odd-mass systems and the rotation alignment process [18]. For odd-proton nuclei in this mass region, it has been suggested [19] that the aligning neutrons are from the  $h_{11/2}$  orbital which is included in the calculation (see below). It should be noted that in the present case of triaxial deformation, any qp-state is a superposition of all possible  $K$ -values. The rotational bands with the triaxial basis states (1) are obtained by specifying different values for the  $K$ -quantum number in the projection operator, Eq. (2). The allowed values of the  $K$ -quantum number for a given intrinsic state are obtained through the following symmetry requirement. For  $\hat{S} = e^{-i\pi\hat{J}_z}$ , we have

$$\hat{P}_{MK}^I |\Phi\rangle = P_{MK}^I \hat{S}^\dagger \hat{S} |\Phi\rangle = e^{i\pi(K-\kappa)} P_{MK}^I |\Phi\rangle, \quad (3)$$

where  $\hat{S} |\Phi\rangle = e^{-i\pi\kappa} |\Phi\rangle$ . It is easy to see that for the self-conjugate vacuum or 0-qp state,  $\kappa = 0$  and, therefore, it follows from Eq. (3) that only  $K = \text{even}$  values are permitted for this state. For two-qp states, the possible values for  $K$ -quantum number are both even and odd, depending on the structure of the qp state. For one-qp state,  $\kappa = 1/2$  ( $-1/2$ ), and the possible values of  $K$  are therefore  $1/2, 5/2, 9/2, \dots$  ( $3/2, 7/2, 11/2, \dots$ ) that satisfy Eq. (3).

As in the earlier PSM calculations, we use the quadrupole-quadrupole plus pairing Hamiltonian [17]

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi \sum_{\mu} \hat{Q}_{\mu}^{\dagger} \hat{Q}_{\mu} - G_M \hat{P}^{\dagger} \hat{P} - G_Q \sum_{\mu} \hat{P}_{\mu}^{\dagger} \hat{P}_{\mu}. \quad (4)$$

The corresponding triaxial Nilsson mean-field Hamiltonian is given by

$$\hat{H}_N = \hat{H}_0 - \frac{2}{3}\hbar\omega \left\{ \epsilon \hat{Q}_0 + \epsilon' \frac{\hat{Q}_{+2} + \hat{Q}_{-2}}{\sqrt{2}} \right\}, \quad (5)$$

where  $\epsilon$  and  $\epsilon'$  specify the axial and triaxial deformations, respectively. In the above equations,  $\hat{H}_0$  is the spherical single-particle Hamiltonian, which contains a proper spin-orbit force. The interaction strengths are taken as follows: The  $QQ$ -force strength  $\chi$  in Eq. (4) is adjusted such that the physical quadrupole deformation  $\epsilon$  is obtained as a result of the self-consistent mean-field calculation [17]. The monopole pairing strength  $G_M$  is of the standard form:  $G_M = [20.25 \mp 16.20(N-Z)/A]/A$ , with “ $-$ ” for neutrons and “ $+$ ” for protons. This choice of  $G_M$  is appropriate for the single-particle space employed in the present calculation, where three major oscillation shells are used for each type of nucleons ( $N = 3, 4, 5$  for neutrons and  $N = 2, 3, 4$  for protons). The quadrupole pairing strength  $G_Q$  is assumed to be proportional to

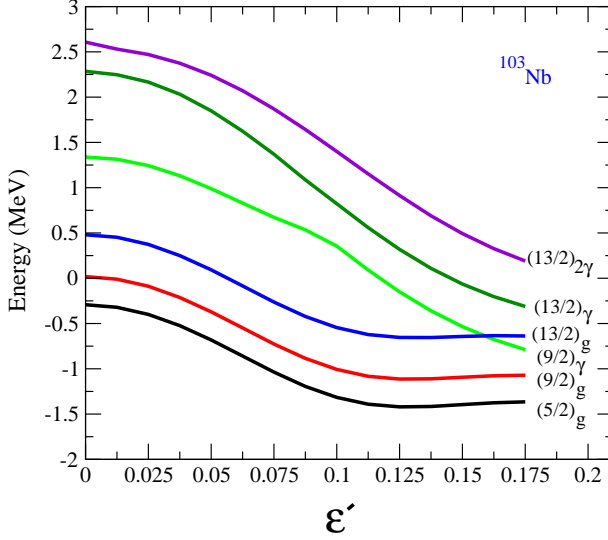


Fig. 1. (Color online) Variation of the projected energy surfaces as functions of triaxiality  $\epsilon'$ . Shown in the plot are three members of the  $K = 5/2$  ground band (denoted with a sub-index “g”), two members of the  $K = 9/2$   $\gamma$ -band (denoted with a sub-index  $\gamma$ ), and one state of the  $K = 13/2$   $\gamma\gamma$ -band (denoted with a sub-index  $2\gamma$ ).

$G_M$ , the proportionality constant being fixed as usual to be 0.16 [17]. These interaction strengths are, although not exactly the same, consistent with those used earlier in the PSM calculations.

Calculations have been performed for  $^{103}\text{Nb}$  with the quadrupole deformation parameter  $\epsilon = 0.3$  and the triaxial one  $\epsilon' = 0.16$ . The value of  $\epsilon$  has been chosen from the total-routhian-surface (TRS) calculations based on the cranked shell model approach with Woods-Saxon potential and Strutinsky shell correction formalism. This value has also been employed in the analysis of the experimental data in Ref. [7]. Nuclei in this mass region are known to exhibit triaxiality [20]. The value of  $\epsilon'$  in our calculation has been chosen such that the bandhead energy of the experimental  $\gamma$ -band is reproduced, which will be seen later. In some of our earlier studies for triaxially deformed nuclei,  $\epsilon'$  was fixed from the minimization of the ground state energy as a function of this parameter (see, for example, Ref. [15]). For the present case, we have also calculated ground state energy as a function of  $\epsilon'$ , as shown in Fig. 1, where states belonging to the ground band are denoted with a sub-index “g” and those to  $\gamma$ -band with a sub-index  $\gamma$ . As one can clearly see from the figure, the energy curves of the

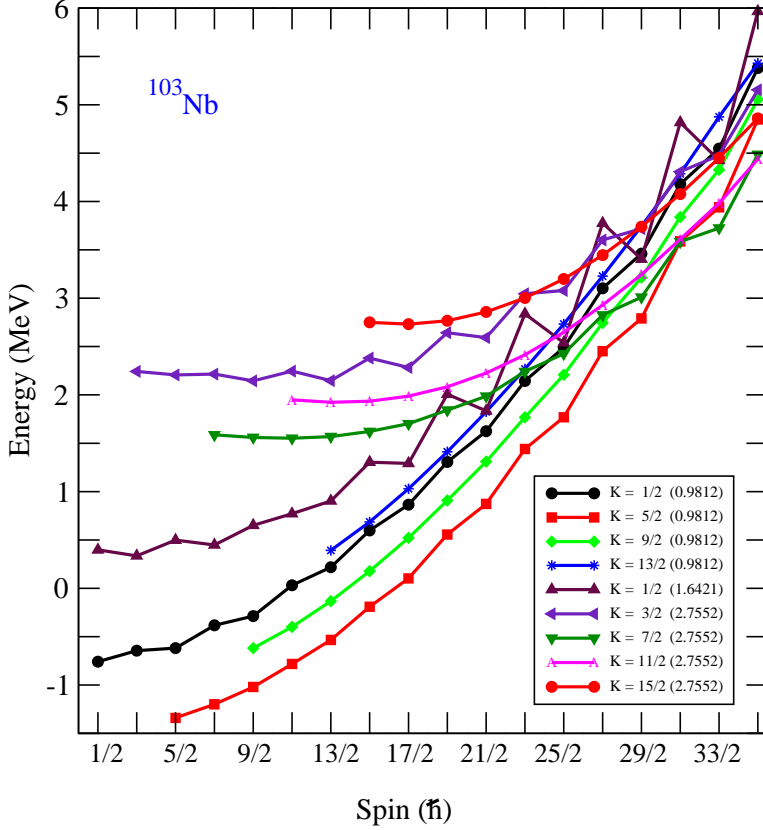


Fig. 2. (Color online) Band diagram for  $^{103}\text{Nb}$  depicting the angular-momentum projected bands from one- and three-quasiparticle states. For clarity, only the lowest projected bands are shown and in the numerical calculations, projection has been performed from forty-four intrinsic states.

states of the ground band are flat in the region of  $\epsilon' = 0.12 - 0.17$ , indicating that these states are very soft with respect to triaxiality. In contrast, energies of the states of the  $\gamma$ -band depend sensitively on  $\epsilon'$ , showing low excitation energies for large triaxiality. This picture thus suggests that the nucleus does not have a well-defined triaxial deformation in its ground state configuration. The same picture has been noted in some even-even nuclear systems [12,21]. Therefore,  $\epsilon'$  in the present treatment is a chosen parameter that reproduces the experimental bandhead of the  $\gamma$ -band and is equal to 0.16. It is pertinent to mention that for the axially symmetric case ( $\epsilon' = 0$ ),  $\gamma$ -bandhead energy is calculated to be 1.63 MeV above the ground-state and the corresponding experimental value is 0.73 MeV.  $\epsilon' = 0.16$  leads to the  $\gamma$ -bandhead energy of 0.72 MeV, which almost reproduces the experimental bandhead energy.

The TPSM analysis is performed in two stages: In the first stage, a set of quasiparticle states are chosen for which the angular-momentum-projection is to be performed. In the second stage, the shell model Hamiltonian is diagonalized in the projected states. In the present work, we have considered an energy window of 3.5 MeV around the Fermi surfaces for both protons and

neutrons, and qp states of (1) with energy smaller than the window value will be selected to form the model basis. As mentioned earlier, the possible values for one-qp state are  $K = 1/2, 5/2, 9/2, \dots$  and the projection from all these  $K$ -states has been performed. In the present calculation it is found that for protons, the lowest one-qp state has an energy of 0.9812 MeV and projection from  $K = 5/2$  of this configuration is the main component of the ground state band. We note that this band depicts a staggering for higher angular-momentum states. This lowest band and the other low-lying excited bands are shown in the band diagram, Fig. 2.

The projection from  $K = 9/2$  with the same lowest one-qp state is the main component of the  $\gamma$ -band built on the ground state of  $K = 5/2$ , and the projection with  $K = 13/2$  gives rise to the  $\gamma\gamma$ -band. The calculated  $\gamma$ - and  $\gamma\gamma$ -bandheads are at excitation energies of 0.73 and 1.73 MeV, respectively. These bandhead energies will be modified by diagonalization, i.e., configuration mixing. It is noted from Fig. 2 that projection from the lowest qp state with  $K = 1/2$  lies higher in energy than the  $\gamma$ -band with  $K = 9/2$ . In general, in odd-A nuclei, there exist two one-phonon  $\gamma$ -bands with  $K_\gamma = K \pm 2$ , where  $K$  is the bandhead quantum number of the quasiparticle state. The excitation energy of  $K - 2$  state is larger than that of  $K + 2$  state [22,7], and in the present analysis corresponds to the  $K = 1/2$  band.

The projection from the qp state with an energy of 1.642 MeV for  $K = 1/2$  is also shown in Fig. 2 as this is a low-lying band. We have also carried out projection from this qp state for other possible values of  $K$ , but they are not shown as these lie at higher excitation energy. It needs to be pointed out that all these states, though not shown here, are employed in the diagonalization of the shell model Hamiltonian.

For three-qp states, one-proton qp state can be coupled with two-neutron states which have  $\kappa = 0$  and 1 and, therefore, three-qp states can also have all possible  $K$ -values. In Fig. 2, only those three-qp bands with lower qp energies are plotted, although the projection has been performed for all other possible  $K$ -values. In the present calculation, the lowest three-qp intrinsic state has an excitation energy of 2.755 MeV, and the projection from this three-qp state with  $K = 7/2$  forms the lowest three-qp band. This three-qp band is the two-neutron aligned band built on the one-qp band having  $K = 5/2$ . It has already been pointed out in our earlier publications [14,15] that each qp state has  $\gamma$ -bands built on it. The  $\gamma$ - and  $\gamma\gamma$ -bands built on the three-qp state with energy of 2.755 MeV have  $K = 11/2$  and  $15/2$ , respectively, which are also depicted in Fig. 2. It is evident from the figure that the three-qp band crosses the ground state band at  $I = 31/2$  and above this spin value, the yrast band has mainly a three-qp structure. It is quite interesting to note from Fig. 2 that  $\gamma$ - and  $\gamma\gamma$ -band built on the three-qp band also cross the corresponding one-qp bands.

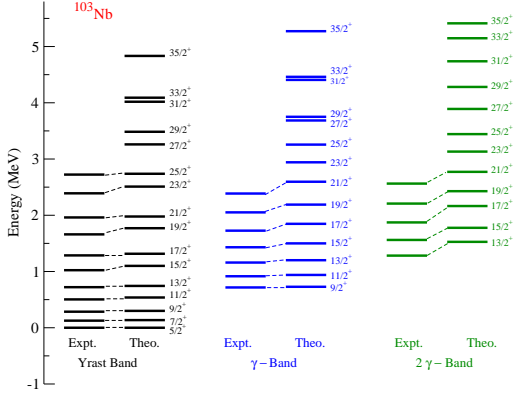


Fig. 3. (Color online) The calculated yrast band,  $\gamma$ - and  $\gamma\gamma$ -bands are compared with the corresponding experimental data [7].

In the second stage of the present calculation, the projected states obtained from a total of forty-four intrinsic states are used as basis for diagonalizing the shell model Hamiltonian (4). The final energy levels after diagonalization are displayed in Fig. 3 for the yrast-,  $\gamma$ - and  $2\gamma$ -bands. This figure also depicts the corresponding experimental band structures obtained in Ref. [7]. It is quite evident from Fig. 3 that TPSM describes the yrast- and  $\gamma$ -bands remarkably well. For the  $\gamma\gamma$ -band, the TPSM predicted band lies higher by about 200 keV as compared to the experimental  $\gamma\gamma$ -band.

The experimental data for  $^{103}\text{Nb}$  points towards anharmonic  $\gamma$ -vibration with the ratio of the bandhead energies of  $\gamma\gamma$ - and  $\gamma$ -band,  $E_{\gamma\gamma}/E_{\gamma} = 1.87$ . The TPSM on the other hand predicts a harmonic  $\gamma$ -vibration with  $E_{\gamma\gamma}/E_{\gamma} = 2.01$ . Of course, the TPSM prediction depends on the choice of the input parameter  $\epsilon'$ , which can be clearly seen from Fig. 1. A smaller  $\epsilon'$  than the used value of 0.16 will produce anharmonicity. However, a smaller  $\epsilon'$  worsens the achieved agreement of the  $\gamma$ -band. The reason for this difficulty could be traced to the  $\gamma$ -softness of this nucleus, evident from the energy-surface calculations in Fig. 1. In a more accurate treatment, the projection needs to be performed for different values of  $\epsilon'$  and then admix these projected states using the generator coordinated method (GCM). We expect that this kind of analysis may possibly explain the anharmonic  $\gamma$ -vibration observed in  $^{103}\text{Nb}$ .

In conclusion, recent experimental data obtained by the spontaneous fission of  $^{252}\text{Cf}$  have found  $\gamma$ - and  $\gamma\gamma$ -bands in the odd-proton nucleus  $^{103}\text{Nb}$ . Motivated by this, we have extended the triaxial projected shell model approach



to odd-mass systems with the inclusion of three-dimensional projected one- and three-qp configurations in the shell model space. As a first application, the band structures of  $^{103}\text{Nb}$  have been investigated. It has been demonstrated that the observed yrast- and  $\gamma$ -bands are reproduced quite well by the TPSM approach, thus supporting the interpretation of the data. However, for the  $\gamma\gamma$ -band, there appears an overall shift in the energies, resulting a more harmonic phonon vibration picture than suggested by data. We have proposed that this discrepancy could possibly be resolved by performing GCM calculations with triaxial deformation  $\epsilon'$  as the generator coordinate.

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